

Phenomenological studies in QCD resummation

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We study applications of QCD soft-gluon resummations to electroweak annihilation cross sections. We focus on a formalism that allows to resum logarithmic corrections arising near partonic threshold and at small transverse momentum simultaneously.

1. INTRODUCTION

When probed near an exclusive boundary of phase space, perturbative partonic hard-scattering cross sections for electroweak-boson (γ^* , W , Z , H) production acquire large logarithmic corrections arising from incomplete cancellations of soft-gluon effects between virtual and real diagrams. The two prominent examples are *threshold* and *recoil* corrections. The former are of the form $\alpha_s^n \ln^{2n-1}(1-z)/(1-z)$ and become large when the partonic c.m. energy approaches the invariant mass Q of the produced boson, $z = Q^2/\hat{s} \rightarrow 1$. The recoil corrections, in turn, are of the form $\alpha_s^n \ln^{2n-1}(Q^2/Q_T^2)$ and grow large if the transverse momentum carried by the produced boson is very small, $Q_T \ll Q$. Therefore, sufficiently close to the phase-space boundary, i.e. in the limit of soft and/or collinear radiation, fixed-order perturbation theory is bound to fail. A proper treatment of the cross section requires resummation of the logarithmic corrections to all orders. The techniques for this are well established in both the threshold [1,2] and in the recoil [3–6] cases.

Resummation of recoil and threshold corrections, however, is known to lead to opposite effects – suppression and enhancement of the partonic cross section, respectively. A full analysis of soft gluon effects in transverse momentum distributions $d\sigma/dQ^2 dQ_T^2$ should therefore, if possible, take both types of corrections simultaneously into account. A joint treatment of the threshold and recoil corrections was proposed in [7,8]. It relies on a novel refactorization of short-distance and long-distance physics at fixed transverse momentum and energy [8]. Similarly to standard threshold and recoil resummations, exponentiation

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of logarithmic corrections occurs in the impact parameter b space, Fourier-conjugated to transverse momentum Q_T space, and Mellin- N moment space, conjugated to z space. This time both transforms are present, resulting in a final expression which obeys energy and transverse-momentum conservation. Consequently, phenomenological evaluation of the joint resummation expressions requires prescriptions for inverse transforms from both N and b spaces. This issue is also closely tied to specifying the border between resummed perturbation theory and the nonperturbative regime, through analysis of the nonperturbative effects implied by the resummation formula itself. Moreover, to fully define the expressions a procedure for matching between the fixed-order and the resummed result needs to be specified. A full phenomenological study of the joint resummation formalism as applied to vector boson production was undertaken in [9]. The formalism may also be applied to Higgs production via gluon-gluon fusion [10]. In this case the Higgs-gluon interaction proceeds through a top quark loop and may, for $m_t > m_h$, be replaced [11] by a simple effective ggh vertex. In the following we will briefly discuss our results for joint resummation as applied to electroweak-boson production.

2. THE JOINTLY RESUMMED CROSS SECTION

In the framework of joint resummation, the resummed electroweak annihilation cross section has the following form [8,9]:

$$\frac{d\sigma_{AB}^{\text{res}}}{dQ^2 dQ_T^2} = \sum_a \sigma_a^{(0)}(Q^2) \int_{C_N} \frac{dN}{2\pi i} \tau^{-N} \int \frac{d^2b}{(2\pi)^2} e^{i\vec{Q}_T \cdot \vec{b}} \times C_{a/A}(Q, b, N, \mu, \mu_F) \exp \left[E_{a\bar{a}}^{\text{PT}}(N, b, Q, \mu) \right] C_{\bar{a}/B}(Q, b, N, \mu, \mu_F), \quad (1)$$

where $\sigma_a^{(0)}(Q^2)$ denotes a perturbative normalization that only depends on the large invariant mass Q of the produced boson [9,10]. We have defined $\tau = Q^2/S$. The flavor-diagonal exponent $E_{a\bar{a}}^{\text{PT}}$ was derived in [9] to next-to-leading logarithmic (NLL) accuracy:

$$E_{a\bar{a}}^{\text{PT}}(N, b, Q, \mu, \mu_F) = - \int_{Q^2/\chi^2}^{Q^2} \frac{dk_T^2}{k_T^2} \left[A_a(\alpha_s(k_T)) \ln \left(\frac{Q^2}{k_T^2} \right) + B_a(\alpha_s(k_T)) \right]. \quad (2)$$

It has the classic form of the Sudakov exponent in the recoil-resummed Q_T distribution for electroweak annihilation, with the A and B functions defined as perturbative series in α_s [3–6]. The quantity $\chi(N, b)$ organizes the logarithms of N and b in joint resummation [9]:

$$\chi(\bar{N}, \bar{b}) = \bar{b} + \frac{\bar{N}}{1 + \bar{b}/4\bar{N}}, \quad (3)$$

where we define $\bar{N} = Ne^{\gamma_E}$, $\bar{b} = bQe^{\gamma_E}/2$, with γ_E the Euler constant. With this choice for $\chi(\bar{N}, \bar{b})$ the LL and NLL terms are correctly reproduced in the threshold limit, $N \rightarrow \infty$ (at fixed b), and in the recoil limit $b \rightarrow \infty$ (at fixed N).

The coefficients in the expansions of the functions in (2) are the same as in the pure Q_T resummation and are known from comparison with fixed-order calculations [12–15] for both vector-boson and Higgs production. At NLL only $A^{(1)}$, $B^{(1)}$ and $A^{(2)}$ contribute,

$$A_a^{(1)} = C_a, \quad A_a^{(2)} = \frac{C_a}{2} \left[C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{10}{9} T_R N_F \right], \quad (C_q = C_F, \quad C_g = C_A),$$

$$B_q^{(1)} = -\frac{3}{2}C_F, \quad B_g^{(1)} = -\frac{1}{6}(11C_A - 4T_R N_F). \quad (4)$$

The second-order term $B_g^{(2)}$ contributes only at NNLL level. It was noted in a previous study on Q_T resummation for Higgs production [16] that the contribution from $B_g^{(2)}$ is actually numerically rather significant due to the size of C_A , and we therefore include it in our study despite the fact that it is subleading to our analysis. We also note that there is an interplay [17] between $B_g^{(2)}$, the function $\sigma_a^{(0)}(Q^2)$ above, and the coefficients $C_{a/H}$ to be specified below; for details, see [17]. We will use in the case of Higgs production [15,17]

$$B_g^{(2)} = C_A^2 \left(-\frac{4}{3} + \frac{11}{36}\pi^2 - \frac{3}{2}\zeta_3 \right) + \frac{1}{2}C_F T_R N_F + C_A N_F T_R \left(\frac{2}{3} - \frac{\pi^2}{9} \right). \quad (5)$$

The functions $\mathcal{C}(Q, b, N, \mu, \mu_F)$ in Eq. (1) are given as:

$$C_{a/H}(Q, b, N, \mu, \mu_F) = \sum_{j,k} C_{a/j}(N, \alpha_s(\mu)) \mathcal{E}_{jk}(N, Q/\chi, \mu_F) f_{k/H}(N, \mu_F). \quad (6)$$

They are products of parton distribution functions $f_{k/H}$ at scale μ_F , an evolution matrix \mathcal{E}_{jk} , and coefficients $C_{a/j}(N, \alpha_s)$ which are perturbative series in α_s . Explicit expressions for the latter are given in [9,10]. The matrix $\mathcal{E}(N, Q/\chi, \mu_F)$ represents the evolution of the parton densities from scale μ_F to scale Q/χ up to NLL accuracy [9] in $\ln N$. By incorporating full evolution of parton densities the cross section (1) correctly includes the leading $\alpha_s^n \ln^{2n-1}(\bar{N})/N$ collinear non-soft terms to all orders. Such terms were previously addressed in [18]. In fact, due to our treatment of evolution, expansion of the resummed cross section (1) in the limit $N \rightarrow \infty$, $b = 0$ gives all $\mathcal{O}(1/N)$ terms in agreement with the $\mathcal{O}(\alpha_s)$ result. Further comparison can be undertaken in the limit $b \rightarrow \infty$, $N = 0$ when our joint resummation turns into standard Q_T resummation. Also, a numerical comparison [9,10] between the fixed-order and the $\mathcal{O}(\alpha_s)$ -expanded jointly resummed expression for $d\sigma/dQ_T$ at shows very good agreement, especially at small Q_T .

3. INVERSE TRANSFORMS AND MATCHING

The jointly resummed cross section (1) requires defining inverse Mellin and Fourier transforms so that singularities associated with the Landau pole are avoided. A contour for the Mellin integral in (1) is chosen in analogy with the ‘minimal prescription’ contour in threshold resummation [19]:

$$N = C + ze^{\pm i\phi}, \quad (7)$$

where the constant C lies to the right of the rightmost singularity of the parton distribution functions but left of the Landau pole.

The inverse Fourier integral from b space also suffers from the Landau singularity. We define this integral with a similar strategy. We first use the identity

$$\int d^2b e^{i\vec{q}\cdot\vec{b}} f(b) = 2\pi \int_0^\infty db b J_0(bq) f(b) = \pi \int_0^\infty db b [h_1(bq, v) + h_2(bq, v)] f(b), \quad (8)$$

and employ Cauchy’s theorem to deform the integration over real b into a contour in the complex b plane [7,9]. Here the auxiliary functions $h_{1,2}$ are related to Hankel functions.

They distinguish between the positive and negative phases in Eq. (8). The b integral can thus be written as a sum of two contour integrals, over the integrand with h_1 (h_2) along a contour in the upper (lower) half of the b plane. The precise form of the contours becomes unimportant as long as the contours do not run into the Landau pole or into singularities associated with the particular form (3) of the function χ . Our treatment of contours in complex transform b -space is completely equivalent to the original form, Eq. (8), when the exponent is evaluated to finite order in perturbation theory. In the presence of the Landau pole arising in the resummed formula, it is a natural extension of the N -space contour redefinition above [19], using a generalized “minimal” exponent. We emphasize that joint resummation with its contour integration method provides an alternative to the standard b space resummation. Joint resummation has built-in a perturbative treatment of large b values, eliminating the need for a b_* or other prescription for the exponent, or for a freezing of the scale of parton distributions at large b or low Q_T . In this way, we can derive entirely perturbative resummed cross sections.

In the joint resummation we adopt the following matching prescription between the resummed and the fixed-order result:

$$\frac{d\sigma}{dQ^2 dQ_T^2} = \frac{d\sigma^{\text{res}}}{dQ^2 dQ_T^2} - \frac{d\sigma^{\text{exp}(k)}}{dQ^2 dQ_T^2} + \frac{d\sigma^{\text{fixed}(k)}}{dQ^2 dQ_T^2}, \quad (9)$$

where $d\sigma^{\text{res}}/dQ^2 dQ_T^2$ is given in Eq. (1) and $d\sigma^{\text{exp}(k)}/dQ^2 dQ_T^2$ denotes the terms resulting from the expansion of the resummed expression in powers of $\alpha_s(\mu)$ up to the order k at which the fixed-order cross section $d\sigma^{\text{fixed}(k)}/dQ^2 dQ_T^2$ is taken. The above matching prescription in (N, b) space guarantees that no double counting of singular contributions occurs in the matched distribution.

4. NUMERICAL RESULTS

Joint resummation predictions for Z boson production compared with the latest CDF data from the Tevatron collider [20] are shown in Fig. 1. Fig. 2 shows our results for the jointly resummed cross section for the production of a 125 GeV Higgs boson at the LHC. Due to the contour integral prescription for performing inverse transforms, in the framework of joint resummation one does not require any extra nonperturbative information to obtain predictions. This is not the case in the standard Q_T resummation formalism, where nonperturbative parameters are introduced to make the theoretical expression well defined.

As shown by the dashed line in Fig. 1 the joint resummation without any extra nonperturbative input already provides a good description of the data for Z production, except for the region of very small Q_T , where the nonperturbative effects are expected to play a significant role. However, the form of the nonperturbative input can be predicted within the joint resummation by taking the limit of small transverse momentum of soft radiation in the exponent, Eq. (2). Assuming moderate threshold effects the procedure gives a simple Gaussian parametrization $F_{NP}(b) = \exp(-gb^2)$. The value of the parameter $g = 0.8\text{GeV}^2$ is determined by fitting the predicted distribution to the data. It is very similar to the value obtained in Ref. [21], where an extrapolation of the exponent to large b was carried out for the Q_T -resummed cross section. The solid line in Fig. 1 represents

predictions including the nonperturbative parametrization. In the large Q_T region, see Fig. 1b, the joint resummation formalism with the matching prescription (9) also returns a very good description of data without requiring an additional switching to a pure fixed-order result, unlike in the standard Q_T resummation formalism. Nevertheless, at large Q_T , no formalism based on the resummation of Sudakov logarithms can be expected to incorporate all relevant contributions, particularly at small x . The relations between Q_T resummation and threshold and joint resummation in the context of Higgs production at the LHC should shed light on this issue.

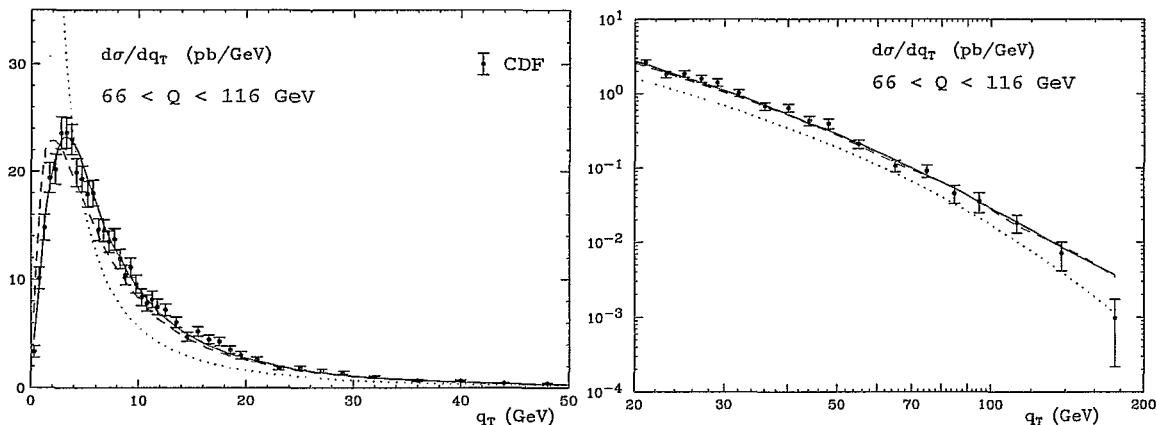


Figure 1. CDF data [20] on Z production compared to joint resummation predictions, without nonperturbative smearing (dashed) and with Gaussian smearing (solid, see text). The dotted line shows the fixed-order result. The normalizations of the curves (factor of 1.035) have been adjusted in order to give an optimal description. We use CTEQ5M [22] parton distribution functions, $\mu = \mu_F = Q$ and $\phi = \phi_b = 25/32\pi$, $C = 1.3$, $b_c = 0.2/Q$.

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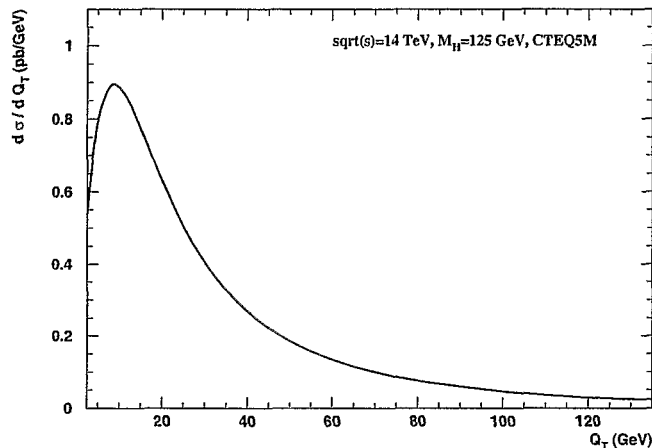


Figure 2. Transverse momentum distribution for Higgs production at the LHC in the framework of joint resummation. We have not implemented any nonperturbative smearing. Parton distributions and other parameters are as in Fig. 1.

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